

U.G. 1st Semester Examination - 2024

PHYSICS

[MINOR]

Course Code : PHY-MI-T-01

(Mathematical Physics-I)

[NEP-2020]

Full Marks : 30

Time : 2 Hours

*The figures in the right-hand margin indicate marks.**Candidates are required to give their answers in their own words as far as practicable.*

GROUP-A

1. Answer any five questions: 1×5=5

- a) State and explain the Green's theorem.
- b) Write down the conditions for a function $f(x)$ to be differentiable at every point.

Check whether the function $f(x) = \frac{1}{(x-x_0)}$ is differentiable at every point in x .

- c) Show that $\vec{A} \cdot \frac{d\vec{A}}{dt} = A \frac{dA}{dt}$.

[Turn over]

- d) Write down the degree and order of the differential equation

$$\frac{d^3 y}{dx^3} = \sqrt{1 + \frac{d^2 y}{dx^2} + \frac{dy}{dx} + \left(\frac{dy}{dx}\right)^2}$$

- e) Find the inverse of a matrix $A = \begin{pmatrix} a & 0 \\ 0 & d \end{pmatrix}$.
- f) Define the Skew-Hermitian matrix with an example.
- g) Find the value of $x\delta(x-3)$ at the point $x=3$ and $|x|>3$.
- h) Show that $\vec{A}$ and $\vec{B}$ are parallel or anti-parallel if $\vec{A} \times \vec{B} = 0$, where $|\vec{A}|$ and $|\vec{B}|$ are not zero.

GROUP-B

2. Answer any three questions: 5×3=15

- a) If $\vec{r}$ is the position vector of a particle of mass m relative to O and $\vec{F}$ is the external force on the particle then $\vec{\tau} = \vec{r} \times \vec{F}$ is the torque of $\vec{F}$ at O . Show that $\vec{\tau} = \frac{d\vec{L}}{dt}$ where $\vec{L} = \vec{r} \times m\vec{v}$, the angular momentum of the particle.

- b) Verify the Stokes's theorem of $\vec{A} = (2x-y)\hat{i} - yz^2\hat{j} - y^2z\hat{k}$, where S is the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$ and C is its boundary.

- c) Solve the equation $\frac{dy}{dx} = \frac{y^3 + x^2 y}{x^3}$.
- d) A non singular matrix A has eigen values λ_1 and eigenvector $\mathbf{x}^1$. Find the eigenvalues of the inverse matrix A^{-1} .
- e) Show that
$$\delta[(x-x_1)(x-x_2)] = \frac{[\delta(x-x_1) + \delta(x-x_2)]}{|x_1 - x_2|}$$
- f) Show that $y \cos(x) dx + \sin(x) dy = 0$ is an exact differential equation and find its general solution. 2+3

GROUP-C

3. Answer any one question: 10×1=10

- a) i) Solve $\frac{d^2 y}{dx^2} + 9y = \cos(2x)$. 5
- ii) Show that the matrix $U = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix}$ is a unitary matrix. $2\frac{1}{2}$

iii) Prove that the eigenvalues of a hermitian matrix $\mathbf{H}$ are real. $2\frac{1}{2}$

b) i) If a vector $\vec{V} = (x + 2y + az)\hat{i} + (bx - 3y - z)\hat{j} + (4x + cy + 2z)\hat{k}$ is *irrotational* then find the constant a, b, c . 5

ii) Show that $(AB)^\dagger = B^\dagger A^\dagger$, where A and B are two non-commuting matrices. $2\frac{1}{2}$

iii) Write down Stoke's theorem and explain it. $2\frac{1}{2}$
