

U.G. 5th Semester Examination-2024**PHYSICS****[HONOURS]****Discipline Specific Elective (DSE)****Course Code : PHY-H-DSE-T-01****(Advanced Mathematical Physics-I)****[New Syllabus]****Full Marks : 40****Time : $2\frac{1}{2}$ Hours***The figures in the right-hand margin indicate marks.**Candidates are required to give their answers in their own words as far as practicable.*

1. Answer any five questions : $2 \times 5 = 10$
 - i) What do you mean by Norm of a vector?
 - ii) What do you mean by covariant tensor?
 - iii) What is isotropic tensor?
 - iv) Define inverse Laplace transform.
 - v) Suppose I is an ideal in a ring R with an identity element e . Prove that if $e \in I$, then $I = R$.
 - vi) What is symmetric tensor?
 - vii) What do you mean by Cartesian Tensor?
2. Answer any two questions : $5 \times 2 = 10$
 - i) Show that isomorphic vector spaces would have the same dimensions.

[Turn Over]

ii) Find the Laplace transform of $f(t)$ where $f(t) = \frac{\sin(\alpha t)}{t}$, given that, $L\left[\frac{\sin \alpha t}{t}\right] = \tan^{-1}\left(\frac{1}{s}\right)$.

iii) Verify that $A = \begin{pmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 3 & 3 & 3 \end{pmatrix}$ is singular.

iv) If a contravariant tensor of rank two is symmetric in one coordinate system, show that it is symmetric in any coordinate system.

2+3

3. Answer any two questions: 10×2=20

i) a) Show that the vectors $(1,2,1)$; $(2,1,0)$; $(1,-1,2)$ form a basis of the vector space V_3 over the field of real numbers.

b) The vector $\vec{r}$ in three-dimensional space are subject to the transformation $\vec{r}' = \hat{A} \vec{r} = \vec{a} \times \vec{r}$. Show that $\hat{A}$ is a linear operator.

c) Prove that in an n-dimensional vector space every set of (n+1) vectors is linearly dependent. 4+4+2

ii) a) If A_{ij} is skew-symmetric and B_{ij} is symmetric, then show that $B_{ij}A_{ij}=0$.

b) Show that the quadratic form $(D_{ij}x_i x_j)$ is unchanged if D_{ij} is replaced by the symmetric part.

c) Show that the work done per unit volume of the crystal required to produce strain e_i is $\frac{1}{2} c_{ij} e_i e_j$.

d) Define Levi-Civita Symbol. 3+3+2+2

iii) a) Obtain a set of four orthonormal vectors by the Schmidt's method from the vectors $u_1=(1,1,0,1)$, $u_2=(2,0,0,1)$, $u_3=(0,2,3,-2)$, $u_4=(1,1,1,-5)$.

b) Show that the outer product of two contravariant vectors α^i and β^j , $i=1,2,3$, results in a contravariant tensor of rank two. 5+5

iv) a) Find the Laplace transform for a step function.

b) Using Laplace transform solve $\frac{dy}{dt} + y = \sin 3t$, given that $y=0$ when $t=0$.

c) Evaluate: $\int_0^\infty e^{-4t} \frac{\sin^2 t}{t} dt$. 2+5+3